Homework 1

Probability and Statistics

~~Due : Sunday, June 25, 5 pm~~

Due : Monday, June 26, 5 pm

Instructions

Please carefully read and follow the instructions below to complete your homework assignment.

- Ensure that your responses are clear and presented in an organized manner. Submit the questions in the order given in the assignment.

- You may write and scan papers to submit. Do NOT use a black background on the sheets. Alternatively, you can use type the answers too. Latex may be helpful.

- For questions that require mathematical calculations, show all your work and provide detailed explanations wherever necessary.

- For short text response questions, provide clear and concise answers in complete sentences or paragraphs.

- Be aware that some questions are a mix of short text responses and maths.

- There are three questions in total. Start your answer to each of the three main questions in a new page.

- If you believe the approach taken in the hints or solutions is wrong, do reach out. We can discuss and correct it if needed.

1   Minecraft

[4 points]

Minecraft is an open-world voxel game. One required step to beat the game is to obtain *Ender Pearls* which can be done in multiple ways. *Speedrunners* are people who try to beat the game as fast as possible.

In this problem, we will look at how *speedrunners* obtain Ender Pearls, by trading with entities in the game. Each trade attempt will drop random loot, but we will only focus on the trades that give Ender Pearls.

There is a 2% chance of getting an Ender Pearl trade on each attempt (this is considered a favorable/successful trade). A successful trade gives an average of 3 pearls. That is,
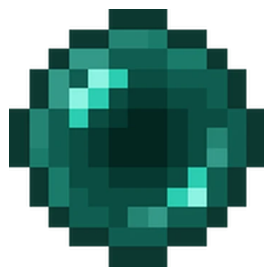
$$P(Trade) = 0.02$$

Figura 1: Ender Pearl

1. Players need a minimum of 12 pearls to complete the game. This means the player will need at least 4 pearl trades to progress. What is the probability that after 270 trade attempts, the player will have received at least 4 favorable trades?

[1 point]

2. On a successful trade, the player receives a uniform distribution between 2-4 pearls (inclusive) What is the probability that, given at least 4 successful trades, the player does **not** receive at least 12 pearls?

[1 point]

Hint: Let T be a random variable with a 2% chance of success, and let E be a random variable with a uniform distribution between 2-4, and it is conditionally dependent on T being successful. So $P(E = 2|T = 1) = 1/3$. It may be helpful to remember that $P(E = x|T \geq y) = P(E = x|T = y)P(T = y|T \geq y) + P(E = x|T > y)P(T > y|T \geq y)$ What is $P(E < 12|T \geq 4)$?

3. Overall, what is the probability that across 360 trade attempts, the player receives at least 12 pearls?

[1 point]

Hint: Let T be a random variable with a 2% chance of success, and let E be a random variable with a uniform distribution between 2-4, and it is conditionally dependent on T being successful. Remember that since E is conditionally dependent on T, $P(E \geq X \cap T = Y) = P(E \geq X|T = Y) * P(T = Y)$. What is $P(E \geq 12)$?
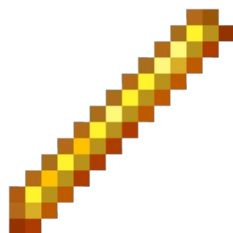

Figura 2: Blaze Rod

4. Another important step for completing the game is obtaining *Blaze Rods*, which have a 50% chance to drop from killing a *Blaze enemy* The player needs 6 blaze rods to progress in the game. *You can assume that collecting Ender Pearls and Blaze Rods are independent.*

Assume the player has seemingly good luck, and gets 6 Ender Pearl trades in the first 90 trade attempts, and then they proceed to get 6 Blaze Rods from the first 8 Blaze enemies killed. What is the probability of this happening? [1 point]

Hint: Let T be a random variable with a 2% chance of success. Let B be another random variable with a 50% chance of success that is independent from T. Given 90 random rolls of T and 8 random rolls of B, what is $P(T = 6 \cap B = 6)$?

2 TV Show Ratings

[5 points]

You are watching two very popular TV shows, *Whose Line of Code Is It Anyway?* and *The Bug Hunters*.

Whose Line of Code Is It Anyway? has 10 episodes with the following viewership - 4M, 3M, 9M, 15M, 1M, 3M,20M, 18M, 3M, and 2.5M. *The Bug Hunters* has 10 episodes with the following viewership - 4M, 3M,4M, 3M, 2.9M, 4M, 3M, 3.5M, 4M, 3.9M

The Bug Hunters has a positive correlation between rating and social media buzz. The show *Whose Line of Code Is It Anyway?*, on the other hand, has a negative correlation between rating and social media buzz.

Hint: Let x and y be a distributions with the following weekly data points (numbers in the millions): $x = [4, 3, 9, 15, 1, 3, 20, 18, 3, 2.5]$ $y = [4, 3, 4, 3, 2.9, 4, 3, 3.5, 4, 3.9]$

1. What is the average viewership of each show for the season? Which show has a higher average viewership, and by how much? [1 point]

   Hint: What is $\bar{x}$ and $\bar{y}$? Which is larger and what is $|\bar{x} - \bar{y}|$?

2. Which show has a higher viewership variance, and what does it indicate about the stability of their viewership?

   [1 point]

   Hint: What is σ_x^2 and σ_y^2? What does it mean to have high or low σ^2?

3. How does the viewership of both shows differ? ie What is the covariance between both shows? [1 point]

   Hint: What is $Cov(x, y)$? What does this mean about the relationship between x and y?

4. In a particular week one of the shows saw a rise in social media buzz.

   Hint: Let z be a new distribution modeling social media buzz. Let x_r and y_r be the ratings of the x and y distributions. z and x_r have a negative correlation. z and y_r have a positive correlation.

   (a) If the show was *Whose Line of Code Is It Anyway?*, is it good news or bad news for the rating of the show? Why?

      [1 point]

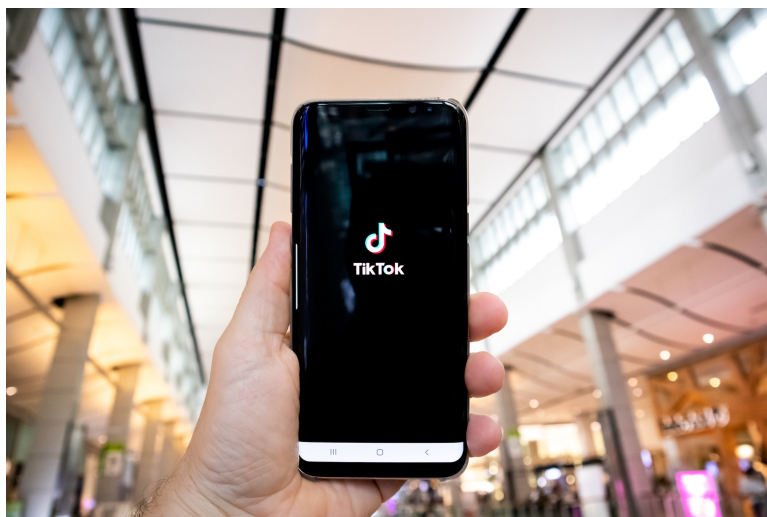   **Hint:** As z increases, what happens to x_r?

   (b) What if it was *The Bug Hunters* instead? Why?

      [1 point]

      Hint: As z increases, what happens to y_r?

3 Influencer Marketing Manager

[11 points]

You are an Influencer Marketing Manager for a consultancy that works with 4 small businesses. A is a travel company, B is a fitness products company, C is in fashion business, and D is a publishing company. Your typical responsibilities include finding influencers, creating campaign ideas, executing influencer contracts, reviewing deliverables, and monitoring the campaign's metrics.

Shyla, a Tiktok Influencer with quite a massive following, has expressed interest in working with you. *Assume that her followers only engage with posts they are interested in.* When we look at her engagement metrics, **70% of the fashion followers** engage with **fashion-related posts**, **80% of the fitness** followers engage with **fitness-related posts**, and **90% of the travel followers** engage with **travel-related posts**.

1. Scenario One   [4 points]

   Assuming her audience consists of **60% followers interested in fashion, 30% followers interested in fitness, and 10% followers interested in travel.** Assume *there is no overlap between these groups; they are all mutually exclusive.*

   Which small business should Shyla work with for maximum engagement? Why?

   Hint: Assume Shyla has 100 fans. How many of her followers are interested in fashion. Of that number, only 70% engage with her posts. How many fans overall engage with her fashion post? Repeat for the other post subjects. Which post subject engages the most people?

2. Scenario Two (DROPPED)   [6 points]
   This question was dropped.