Homework 1

Probability and Statistics

Due : Sunday, June 25, 5 pm

Instructions

Please carefully read and follow the instructions below to complete your homework assignment.

- Ensure that your responses are clear and presented in an organized manner. Submit the questions in the order given in the assignment.
- You may write and scan papers to submit. Do NOT use a black background on the sheets. Alternatively, you can use type the answers too. Latex may be helpful.
- For questions that require mathematical calculations, show all your work and provide detailed explanations wherever necessary.
- For short text response questions, provide clear and concise answers in complete sentences or paragraphs.
- Be aware that some questions are a mix of short text responses and maths.
- There are three questions in total. Start your answer to each of the three main questions in a new page.

1 Minecraft

[4 points]

Minecraft is an open-world voxel game. One required step to beat the game is to obtain *Ender Pearls* which can be done in multiple ways. *Speedrunners* are people who try to beat the game as fast as possible.

In this problem, we will look at how *speedrunners* obtain Ender Pearls, by trading with entities in the game. Each trade attempt will drop random loot, but we will only focus on the trades that give Ender Pearls.

There is a 2% chance of getting an Ender Pearl trade on each attempt (this is considered a favorable/successful trade). A successful trade gives an average of 3 pearls. That is,

$$P(Trade) = 0.02$$

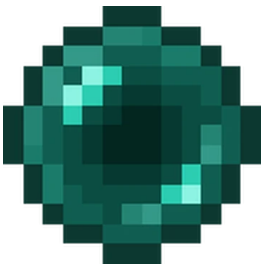

Figura 1: Ender Pearl

1. Players need a minimum of 12 pearls to complete the game. This means the player will need at least 4 pearl trades to progress. What is the probability that after 270 trade attempts, the player will have received at least 4 favorable trades?

[1 point]

Solution:
$P(T = x) = \binom{270}{x} * 0.02^x * 0.98^{270-x}$
$P(T \geq 4) = 1 - P(T < 4)$
$= 1 - \sum_{n=0}^{3} \binom{270}{n} * 0.02^n * 0.98^{270-n}$
$= 0.7896$

2. On a successful trade, the player receives a uniform distribution between 2-4 pearls (inclusive) What is the probability that, given at least 4 successful trades, the player does **not** receive at least 12 pearls?

[1 point]

Solution:
$$\begin{aligned}
P(E < 12|T \geq 4) &= 1 - P(E \geq 12|T \geq 4) \\
&= 1 - (P(E \geq 12|T = 4) * P(T = 4|T \geq 4) \\
&\quad + P(E \geq 12|T = 5) * P(T = 5|T \geq 4) \\
&\quad + P(E \geq 12|T \geq 6) * P(T \geq 6|T \geq 4)) \\
&= 1 - (P(E \geq 12|T = 4) * P(T = 4 \cap T \geq 4)/P(T \geq 4) \\
&\quad + P(E \geq 12|T = 5) * P(T = 5 \cap T \geq 4)/P(T \geq 4) \\
&\quad + P(E \geq 12|T \geq 6) * P(T \geq 6 \cap T \geq 4)/P(T \geq 4)) \\
&= 1 - (P(E \geq 12|T = 4) * P(T = 4)/P(T \geq 4) \\
&\quad + P(E \geq 12|T = 5) * P(T = 5)/P(T \geq 4) \\
&\quad + P(E \geq 12|T \geq 6) * P(T \geq 6)/P(T \geq 4)) \\
&= 1 - (P(E \geq 12|T = 4) * P(T = 4) \\
&\quad + P(E \geq 12|T = 5) * P(T = 5) \\
&\quad + P(E \geq 12|T \geq 6) * P(T \geq 6))/P(T \geq 4)
\end{aligned}$$

$P(E \geq 12|T \geq 6) = 1$. Because we are guaranteed at least 2 Ender Pearls per trade, so at a minimum we will get 12 Pearls from at least 6 trades.

$P(E \geq 12|T = 5) = 1 - P(E < 12|T = 5)$
$P(E < 12|T = 5) = 6/3^5 = 0.0247$

There are 3^5 possible combinations of pearls given from 5 trades, which all have equal probability of occurring due to the uniform distribution. 6 of these combinations have E<12, 1 from getting all 2's, and 5 from getting all 2's except a single 3 in any of the 5 positions.
$P(E \geq 12|T = 5) = 1 - 0.0247 = 0.9753$

$P(E = 12|T = 4) = 19/3^4 = 0.2346$

From: 1 possibility from (3, 3, 3, 3), 6 possibilities from combinations of (2, 2, 4, 4), and 12 possibilities from combinations of (2, 3, 3, 4). And there are 3^4 total possibilities.

$1 = P(E < 12|T = 4) + P(E = 12|T = 4) + P(E > 12|T = 4)$
$= 2 * P(E > 12|T = 4) + P(E = 12|T = 4)$
Since $\mathbb{E}[E|T = 4] = 12$, and there is a uniform distribution,
then $P(E < 12|T = 4) = P(E > 12|T = 4)$.
$P(E > 12|T = 4) = (1 - P(E = 12|T = 4))/2 = 0.3828$
$P(E \geq 12|T = 4) = P(E = 12|T = 4) + P(E > 12|T = 4) = 0.6174$

$P(T = 4) = \binom{270}{4} * 0.02^4 * 0.98^{266} = 0.1606$
$P(T = 5) = \binom{270}{5} * 0.02^5 * 0.98^{265} = 0.1744$
$P(T \geq 6) = 1 - \sum_{i=0}^{5} \binom{270}{i} * 0.02^i * 0.98^{270-i} = 0.4546$

$$\begin{aligned} P(E < 12|T \geq 4) &= 1 - (P(E \geq 12|T = 4) * P(T = 4) \\ &\quad + P(E \geq 12|T = 5) * P(T = 5) \\ &\quad + P(E \geq 12|T \geq 6) * P(T \geq 6))/P(T \geq 4) \\ &= 1 - (0.6174 * 0.1606 + 0.9753 * 0.1744 + 1 * 0.4546)/0.7896 \\ &= 0.0832 \end{aligned}$$

3. Overall, what is the probability that across 360 trade attempts, the player receives at least 12 pearls?

[1 point]

Solution:

$$\begin{aligned} P(E \geq 12) &= P(E \geq 12 \cap T \leq 2) + P(E \geq 12 \cap T = 3) + P(E \geq 12 \cap T = 4) \\ &\quad + P(E \geq 12 \cap T = 5) + P(E \geq 12 \cap T \geq 6) \\ &= P(E \geq 12|T \leq 2) * P(T \leq 2) + P(E \geq 12|T = 3) * P(T = 3) \\ &\quad + P(E \geq 12|T = 4) * P(T = 4) + P(E < 12|T = 5) * P(T = 5) \\ &\quad + P(E \geq 12|T \geq 6) * P(T \geq 6) \\ &= 0 * P(T \leq 2) + 0.0370 * \binom{360}{3} * 0.02^3 * 0.98^{357} \\ &\quad + (1 - P(E < 12|T = 4)) * \binom{360}{4} * 0.02^4 * 0.98^{356} \\ &\quad + (1 - P(E < 12|T = 5)) * \binom{360}{5} * 0.02^5 * 0.98^{355} + 1 * (1 - P(T < 6)) \\ &= 0.0370 * 0.0455 + 0.6172 * 0.0829 + 0.9753 * 0.1204 + 0.726 \\ &= 0.8963 \end{aligned}$$

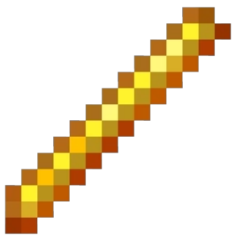

Figura 2: Blaze Rod

4. Another important step for completing the game is obtaining *Blaze Rods*, which have a 50% chance to drop from killing a *Blaze enemy* The player needs 6 blaze rods to progress in the game. *You can assume that collecting Ender Pearls and Blaze Rods are independent.*

   Assume the player has seemingly good luck, and gets 6 Ender Pearl trades in the first 90 trade attempts, and then they proceed to get 6 Blaze Rods from the first 8 Blaze enemies killed. What is the probability of this happening? [1 point]

Solution:

$$\begin{aligned} P(T = 6 \cap B = 6) &= P(T = 6) * P(B = 6) = \binom{90}{6} * 0.02^6 * 0.98^{84} + \binom{8}{6} * 0.5^6 * 0.5^2 \\ &= 0.0073 * 0.1094 = 0.000799 \end{aligned}$$

2 TV Show Ratings

[5 points]

You are watching two very popular TV shows, *Whose Line of Code Is It Anyway?* and *The Bug Hunters*.

Whose Line of Code Is It Anyway? has 10 episodes with the following viewership - 4M, 3M, 9M, 15M, 1M, 3M,20M, 18M, 3M, and 2.5M. *The Bug Hunters* has 10 episodes with the following viewership - 4M, 3M,4M, 3M, 2.9M, 4M, 3M, 3.5M, 4M, 3.9M

The Bug Hunters has a positive correlation between rating and social media buzz. The show *Whose Line of Code Is It Anyway?*, on the other hand, has a negative correlation between rating and social media buzz.

1. What is the average viewership of each show for the season? Which show has a higher average viewership, and by how much? [1 point]

   Solution:
   $\bar{x} = \frac{1}{10}\sum_{i=1}^{10} x_i = 7.85M$
   $\bar{y} = \frac{1}{10}\sum_{i=1}^{10} y_i = 3.53M$
   $\bar{x} > \bar{y}$
   Whose Line of Code Is It Anyway? has higher average viewership by 4.32M.

2. Which show has a higher viewership variance, and what does it indicate about the stability of their viewership?

   [1 point]

   Solution:
   $\sigma_x^2 = \frac{1}{10}\sum_{i=1}^{10} (x_i - \bar{x})^2 = 46.4025M$
   $\sigma_y^2 = \frac{1}{10}\sum_{i=1}^{10} (y_i - \bar{y})^2 = 0.2261M$
   Whose Line of Code Is It Anyway? has less stable viewership since it has higher variance

for weekly views. *The Bug Hunters* has low variance which means it has about the same viewership from week to week.

3. How does the viewership of both shows differ? ie What is the covariance between both shows? [1 point]

   Solution:
   $Cov(x, y) = \frac{1}{10} \sum_{i=1}^{10} (x_i - \bar{x})(y_i - \bar{y}) = -1.1455$
   This shows an inverse relationship between the two shows. When one has a rise in viewership, the other will see a dip (and vice versa).

4. In a particular week one of the shows saw a rise in social media buzz.

   (a) If the show was *Whose Line of Code Is It Anyway?*, is it good news or bad news for the rating of the show? Why?

   [1 point]

   Solution:
   Whose Line of Code Is It Anyway? has a negative correlation between rating and social media buzz. If social media buzz rises, the rating of the show will decrease.

   (b) What if it was *The Bug Hunters* instead? Why?

   [1 point]

   Solution:
   The Bug Hunters has a positive correlation between rating and social media buzz. If social media buzz rises, the rating of the show will increase.

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3 Influencer Marketing Manager

[5 points]

You are an Influencer Marketing Manager for a consultancy that works with 4 small businesses. A is a travel company, B is a fitness products company, C is in fashion business, and D is a publishing company. Your typical responsibilities include finding influencers, creating campaign ideas, executing influencer contracts, reviewing deliverables, and monitoring the campaign's metrics.

Shyla, a Tiktok Influencer with quite a massive following, has expressed interest in working with you. *Assume that her followers only engage with posts they are interested in.* When we look at her engagement metrics, **70% of the fashion followers** engage with **fashion-related posts**, **80% of the fitness** followers engage with **fitness-related posts**, and **90% of the travel followers** engage with **travel-related posts**.

1. Scenario One [5 points]

   Assuming her audience consists of **60% followers interested in fashion, 30% followers interested in fitness, and 10% followers interested in travel.** Assume *there is no overlap between these groups; they are all mutually exclusive.*

   Which small business should Shyla work with for maximum engagement? Why?

   Solution:
   Portion of people who engage with a given post
   = [portion of fanbase] * [portion of engagement for subject]
   Fashion Engagement: 0.6*0.7 = 0.42
   Fitness Engagement: 0.3*0.8 = 0.24
   Travel Engagement: 0.1*0.9 = 0.09
   Shyla should work with fashion brands and post about fashion since it will reach the largest portion of her overall fanbase. 42% of her fans will engage with posts she makes about fashion.

2. Scenario Two (DROPPED) [6 points]